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D and D_S decay constants from QCD duality at three loops*

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Abstract: We compute the decay constants of the pseudoscalar mesons D and D_s using a linear combination of finite energy sum rules which minimize the contribution of the unknown continuum spectral function. We employ the recent three loop calculation of the pseudoscalar two-point function expanded in powers of the running charm quark mass. The theoretical uncertainties arising from the QCD asymptotic expansion are quite relevant in this case due to the relative small scale of the charm mass. We obtain the following results: $f_D = 177 \pm 21\,\mathrm{MeV}$ and $f_{D_s} = 205 \pm 22\,\mathrm{MeV}$. These results, within the error bars, are in good agreement with estimates obtained using Borel transform QCD sum rules, but somewhat smaller than results of recent lattice computations.

Keywords: QCD, Asymptotic Freedom, Sum Rules, NLO Computations.

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1. Introduction

The decay constant of a pseudoscalar meson D_q consisting of a heavy c -quark and a light quark $q = u, d, s$ is defined through the matrix element of the corresponding pseudoscalar current as follows:

$$\langle\Omega|\,(m_c + m_q)\,\overline{q}\,i\,\gamma_5\,c(0)\,|D_q\rangle = \,f_{D_q}\,M^2_{D_q}\,.$$

Recently, two different experimental groups have extracted the decay constant f_{D^+} from a direct measurement of the absolute branching fraction for the Cabibbo-suppressed leptonic decay mode $D^+ \to l^+ v$ [1, 2]. The results are quite different, although there is some overlap of the large experimental errors. Both results are compatible with the upper limit of 290 MeV, established by the MARK-III collaboration at 90% C.L. [3]. For the D_s^+ meson, there are several measurements of its decay constant $f_{D_s^+}$ published in the last decade [4−9], with results in the range of $194 - 430$ MeV. On the theoretical side, there is a recent estimate $f_D = 195 \pm 20$ MeV based on Borel transformed sum rules [10], and a preliminary result $f_D = 225 + 11 - 13 \pm 21$ MeV [11] obtained in three flavor lattice QCD. There appears to be some room for improvement of the accuracy of these results as well as for the systematic study of their uncertainties. On the other hand, the result $f_{D_s}/f_D = 1.18$ for the ratio of the decay constants appears to be well established in lattice QCD [11, 12].

In this letter, we estimate the decay constants f_D and f_{D_s} of the peudoscalar charmed mesons using an alternative method, based on finite energy sum rules, which compare moments of available experimental data with the corresponding QCD theoretical counterpart. In particular, we take linear combinations of positive moments in such a way that the contribution of the data in the region above the resonances turns out to be practically negligible. On the theoretical side we use a large momentum expansion of massive QCD at three loops. This expansion is known up to the seventh power of m_c^2/s, where m_c is the mass of the charm quark and s is the square of the CM energy [13]. The expansion makes sense as long as s is far enough above the continuum threshold and above the resonances. On the phenomenological side of the sum rule we consider only the lowest lying pseudoscalar D_q-meson, once the unknown continuum contribution has been removed by a judicious use of quark-hadron duality in our method [14].

The plan of this note is the following. In section 2 we briefly review the finite energy sum rule method employed. In section 3 we discuss the theoretical and experimental inputs and we present our estimates for the decay constants. Finally, in section 4 we write up the conclusions.

2. The method

The two point function associated with the pseudoscalar current is:

$$\Pi(s=q^2) = i \int dx\, e^{iqx} \langle\Omega|\, T(j_5(x)\, j_5^\dagger(0))\, |\Omega\rangle\,, \tag{2.1}$$

where $\langle\Omega\,|$ is the physical vacuum and the current $j_5(x)$ is the divergence of the axial-vector current:

$$j_5(x) = (M_Q + m_q) : \overline{q}(x)\, i\, \gamma_5\, Q(x) \ : \tag{2.2}$$

M_Q is the mass of the heavy charm quark $Q(x)$ and m_q is the mass of the light quarks $q(x)$, up, down or strange. The starting point of our sum rules is Cauchy's theorem applied to the two-point correlation function $\Pi(s)$, weighted with a polynomial $P(s)$:

$$\frac{1}{2\pi i}\oint_\Gamma P(s)\,\Pi(s)\, ds = 0\,. \tag{2.3}$$

The integration contour Γ extends over a circle of radius $|s| = s_0$, and along both sides of the physical cut $[s_{\rm phys.}, s_0]$ where $s_{\rm phys.}$ is the physical threshold and s_0 is a duality parameter to be fixed with some stability criteria. The polynomial $P(s)$ does not change the analytical properties of $\Pi(s)$. Therefore we obtain the following sum rule:

$$\frac{1}{\pi}\int_{s_{\rm phys.}}^{s_0} P(s)\,\mathrm{Im}\ \Pi(s)\, ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} P(s)\,\Pi(s)\, ds\,. \tag{2.4}$$

On the left hand side of equation (2.4) we use the experimental information for $\mathrm{Im}\,\Pi^{\rm DATA}(s)$ between the physical threshold $s_{\rm phys.}$ and s_0, whereas on the right hand side we use the asymptotic expansion $\Pi^{\rm QCD}(s)$ of QCD, including perturbative and non-perturbative terms. The QCD expansion constitutes a good approximation of the two-point correlate on the circle for a large enough integration radius s_0.

The experimental data are dominated by the first pseudoscalar $c\overline{q}$ resonance. In the narrow width approximation, the absorptive part of the two-point function $\mathrm{Im}\,\Pi^{\rm DATA}(s)$ can be split in two terms, the contribution of the resonance and the contribution of the hadronic continuum $\mathrm{Im}\,\Pi^{\rm cont.}$ starting at the physical continuum threshold $s_{\rm cont.} > s_{\rm phys.} = M_{D_q}^2$, as follows:

$$\frac{1}{\pi}\,\mathrm{Im}\,\Pi^{\rm DATA}(s) = M_{D_q}^4\, f_{D_q}^2\, \delta(s - M_{D_q}^2) + \frac{1}{\pi}\,\mathrm{Im}\,\Pi^{\rm cont.}\,\theta(s - s_{\rm cont.}) \tag{2.5}$$

where M_{D_q} and f_{D_q} are respectively the mass and the decay constants of the lowest lying pseudoscalar meson D_q. Here $s_{cont.}$ is the physical threshold which is fixed by experiment and is not the theshold s_0 where QCD perturbation theory is assumed to hold. The value of s_0 is varied to check for stability.

For the QCD correlate we write the decomposition,

$$\Pi^{\rm QCD}(s) = \Pi^{\rm pert.}(s) \,+\, \Pi^{\rm nonpert.}(s)\,, \tag{2.6}$$

We employ the two-point correlation function $\Pi^{\rm pert.}(s)$ with one massless and one heavy quark given to second order (three loops) in the strong coupling constant α_s and expanded in a power series in the pole mass of the heavy quark including terms of order $(M_c^2/s)^7$. In [13] the following compact expansion of the two-point function in terms of the pole mass M_c can be found

$$\Pi^{\rm pert.}(s) = \Pi^{(0)}(s) + \left(\frac{\alpha_s(M_c)}{\pi}\right)\Pi^{(1)}(s) + \left(\frac{\alpha_s(M_c)}{\pi}\right)^2\Pi^{(2)}(s)\,, \tag{2.7}$$

where the different terms of the expansion in α_s have the form:

$$\Pi^{(i)}(s) = (M_c+m_q)^2 M_c^2 \sum_{j=-1}^{6}\sum_{k=0}^{3} A^{(i)}_{j,k}\left(\ln\frac{-s}{M_c^2}\right)^k\left(\frac{M_c^2}{s}\right)^j. \tag{2.8}$$

In the equations (2.7), (2.8), M_c is the pole mass of the charm quark. The coefficients $A^{(i)}_{j,k}$ are explicitly given in [13]. For instance, the one-loop term of the expansion in α_s reads:

$$\begin{aligned}\Pi^{(0)}(s) \,=\, \frac{3}{16\pi^2}(M_c+m_q)^2 s\Bigg\{&3-2\log\left(\frac{-s}{M_c^2}\right)+4\frac{M_c^2}{s}\log\left(\frac{-s}{M_c^2}\right)-\left[3+2\log\left(\frac{-s}{M_c^2}\right)\right]\times\\ &\times\left(\frac{M_c^2}{s}\right)^2+\frac{2}{3}\left(\frac{M_c^2}{s}\right)^3+\frac{1}{6}\left(\frac{M_c^2}{s}\right)^4+\\ &+\frac{1}{15}\left(\frac{M_c^2}{s}\right)^5+\frac{1}{30}\left(\frac{M_c^2}{s}\right)^6+\frac{2}{105}\left(\frac{M_c^2}{s}\right)^7+\cdots\Bigg\}\end{aligned}$$

The non-perturbative terms in the asymptotic expansion of equation (2.6) are due to the quark and gluon condensates. We will include terms up to dimension six [15, 16]:

$$\begin{aligned}\Pi^{\rm nonpert.}(s) \,=\, (M_c+m_q)^2\Bigg\{&M_c\,\langle\bar{q}q\rangle\,\frac{1}{s-M_c^2}\left[1+2\frac{\alpha_s}{\pi}\left(1+\ln\frac{M_c^2}{-s+M_c^2}\right)\right]-\\ &-\frac{1}{12}\frac{1}{s-M_c^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle-\frac{1}{2}M_c\left[\frac{1}{(s-M_c^2)^2}+\frac{M_c^2}{(s-M_c^2)^3}\right]\times\\ &\times\left\langle\bar{q}\sigma Gq\right\rangle-\frac{8}{27}\pi\left[\frac{2}{(s-M_c^2)^2}+\frac{M_c^2}{(s-M_c^2)^3}-\right.\\ &\left.-\frac{M_c^4}{(s-M_c^2)^4}\right]\alpha_s\left\langle\bar{q}q\right\rangle^2\Bigg\}.\end{aligned}$$

For the quark condensate we include the α_s correction [30], it turns out to be small but non-negligible.

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In order to improve the convergence of the perturbative expansion, we replace the pole mass by the running mass using the $O(\alpha_s^2)$ result relating the two [17, 19, 18, 20]. The perturbative piece of order $(\alpha_s)^i$ of equation (2.8) can be rewritten in the form

$$\Pi^{(i)}(s) = m_c^2(\mu)(m_c(\mu)+m_q(\mu))^2 \sum_{j=-1}^{6}\sum_{k=0}^{3} \tilde{A}_{j,k}^{(i)} \left(\ln\frac{-s}{\mu^2}\right)^k \left(\frac{m_c^2(\mu)}{s}\right)^j \qquad (i=0,1,2)\,. \tag{2.9}$$

and similarly for the non-perturbative piece. The coefficients $\tilde{A}_{j,k}^{(i)}$ depend on the mass logarithms $\ln(m_c^2/\mu^2)$ up to the third power. As $\Pi(s)^{\rm QCD}$ is not known to all orders in α_s, the results of our analysis will depend to some extend on the choice of the renormalization point μ. In the sum rule considered here there are two obvious choices, $\mu = m_c$ and $\mu = \sqrt{s_0}$. The former choice will sum up the mass logs of the form $\ln(m_c^2/\mu^2)$ and the latter choice the $\ln(-s/\mu^2)$ terms. For definiteness, we take $\mu = m_c$. With this, the convergence of the perturbative terms is reasonably good. The results differ from taking $\mu = \sqrt{s_0}$ by an amount consistent with the general three-loop asymptotic uncertainties, as we will analyze below.

Looking back to equation (2.4) and taking all the theoretical parameters as well as the mass of the D_q-meson and the physical continuum threshold as inputs of the calculation, we see that the decay constants can be computed from equation (2.5) only if we have good control over the hadron continuum contribution of the experimental side.

To cope with this problem we make use of the freedom of choosing the polynomial in equation (2.4). We take for $P(s)$ a polynomial of the form:

$$P_n(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots + a_n s^n\,, \tag{2.10}$$

where the coefficients are fixed by imposing a normalization condition at threshold

$$P_n\left(s_{\rm phys.} = M_{D_q}^2\right) = 1\,, \tag{2.11}$$

and requiring that the polynomial $P_n(s)$ should minimize the contribution of the continuum in the range $[s_{\rm cont.}, s_0]$ in a least square sense, i.e.,

$$\int_{s_{\rm cont.}}^{s_0} s^k P_n(s)\, ds = 0 \qquad \text{for } k = 0,\ldots,n-1\,. \tag{2.12}$$

The polynomials obtained in this way are closely related to the Legendre polynomials. In the appendix the explicit form of the set of polynomials used in this work is given.

This way of introducing the polynomial weight in the sum rule minimizes the continuum contribution $\frac{1}{\pi}\,{\rm Im}\,\Pi^{\rm cont.}\,\theta(s - s_{\rm cont.})$ on the phenomenological side of the sum rule. To the extend that ${\rm Im}\,\Pi^{\rm cont.}$ can be approximated by an n-th degree polynomial these conditions lead actually to an exact cancellation of the continuum contribution to the left hand side of equation (2.4). The role of the D_q resonance will be enhanced. In a sense, these polynomials are a generalization of pinched moments. One can easily see from the appendix that our polynomial of degree n approaches $(s - s_0)^n$ when $s_0 \to s_{cont}$. Pinched moments eliminate the contribution on the physical real axis at a single point while our polynomials practically eliminate a whole region (from s_{cont} to s_0).We will see in our analysis that this choice of the polynomial has the additional effect of increasing the region of duality characterized by the value of the duality parameter s_0. In this way the asymptotic expansion of QCD can be used more safely on the circular integration contour. Notice however that increasing the degree of the polynomial $P_n(s)$ will require the knowledge of further terms in the mass expansion and in the non-perturbative series. Therefore the polynomial degree cannot be chosen arbitrarily high.

To check the consistency of the method, we have employed polynomials ranging from second degree to fifth degree, verifying that the results are compatible within the range of the errors introduced by the incomplete knowledge of the QCD correlate and other inputs of the calculation. We also have checked explicitly that a smooth continuum contribution had no influence on the result.

Our approach to suppress the continuum has been tested previously in the calculation of the heavy quark masses using analogue sum rules for the vector current correlate where there exists more experimental information on the continuum. In the calculation of the charm quark mass, using the same polynomial method, the continuum, known from the BES II collaboration [21], was shown to have practically no influence on the predicted mass [23]. Employing the same technique, a very accurate prediction of the bottom quark mass was also obtained using the experimental information of the upsilon system [24].

After these general considerations we proceed with the analytical calculation. The integrals that we have to evaluate on the right hand side of the sum rule, equation (2.4), are

$$J(p,k) = \frac{1}{2\pi i}\oint_{|s|=s_0} s^p \left(\ln\frac{-s}{\mu^2}\right)^k ds\,, \tag{2.13}$$

for $k = 0,1,2,3$ and $p = -6,-5,\ldots,n+1$. These integrals can be found e.g. in reference [22]. After integration, equation (2.4) yields the sum rule

$$\begin{aligned} M_{D_q}^4 f_{D_q}^2 P(M_{D_q}^2) = {} & m_c^2(\mu)(m_c(\mu)+m_q(\mu))^2 \times \\ & \times \sum_{p=0}^{n}\sum_{i=0}^{2}\sum_{j=-1}^{6}\sum_{k=0}^{3} a_p \left(\frac{\alpha_s(\mu)}{\pi}\right)^i \tilde{A}_{j,k}^{(i)}\, m_c^{2j}(\mu)\, J(p-j,k) + \\ & + \text{non-perturbative terms} \end{aligned} \tag{2.14}$$

where, for brevity, we have not written down the non-perturbative terms explicitly. The contribution of the continuum is neglected as explained above although the continuum threshold is considered in the determination of the coefficients, a_p of the polynomials (2.12).

Plugging the theoretical and experimental inputs (physical threshold, quarks and meson masses, condensates and strong coupling constant) into the sum rule, we obtain the decay constant f_{D_q} for various values of the degree n of the polynomial and various values of s_0. Given the correct QCD asymptotic correlate and the correct hadron continuum, the calculation of the decay constant should, of course, not depend either on s_0 or on the degree n of the polynomial in the sum rule (2.4). Accordingly, for a given n we choose the flattest region in the curve $f_{D_q}(s_0)$ to extract our prediction for the decay constant. To be specific we choose the point of minimal slope. On the other hand, for different polynomials, the value of f_{D_q}, extracted in this way, could differ from each other as the cancellation of the continuum may be incomplete or the QCD expansion may not be accurate enough. We find, however, practically the same results for all our polynomials. This additional stability is truly remarkable as the coefficients of the polynomials of order $n = 2, 3, 4$ and 5 are completely different and the respective predictions are based on completely different superpositions of finite energy moment sum rules. This extended consistency leads us to attach great confidence in our numbers and associated errors.

3. Results

We calculate the decay constants for the D and D_s heavy mesons. In the first case we take $m_q = 0$ everywhere. In the second case we retain $m_q = m_s \neq 0$ in the factor $(M_c + m_q)^2$ in front of the correlation function only. Further terms in the power series in m_s^2/s in (2.8) are completely negligible for the integration radii s_0 we use in the calculation.

The experimental and theoretical inputs are as follows. The physical threshold $s_{\mathrm{phys.}}$ is the squared mass of the lowest lying resonance in the $c\overline{q}$ channel. For q being the light quark u, we have:

$$s_{\mathrm{phys.}} = M_D^2 = 3.493\,\mathrm{GeV}^2 \tag{3.1}$$

whereas the continuum threshold $s_{\mathrm{cont.}}$ is taken from the next intermediate state $D\pi\pi$ in an s-wave $I = \frac{1}{2}$, i.e.

$$s_{\mathrm{cont.}} = (M_D + 2m_\pi)^2 = 4.575\,\mathrm{GeV}^2\,.$$

For q being the strange quark we take:

$$s_{\mathrm{phys.}} = M_{D_s}^2 = 3.873\,\mathrm{GeV}^2\,. \tag{3.2}$$

The continuum threshold starts in this case at the value:

$$s_{\mathrm{cont.}} = (M_{D_s} + 2m_\pi)^2 = 5.009\,\mathrm{GeV}^2\,.$$

On the theoretical side of the sum rule we take the following inputs. The strong coupling constant at the scale of the electroweak Z boson mass [27]

$$\alpha_s(M_Z) = 0.118 \pm 0.003 \tag{3.3}$$

that is run down to the renormalization scale using the four loop formulas of reference [26]. For the quark and gluon condensates (see for example [30]) and the mass of the strange quark [28] we take:

$$\begin{aligned}
\langle\overline{q}q\rangle(2\,\mathrm{GeV}) &= (-267\,\pm\,17\,\mathrm{MeV})^3\,,\\
\left\langle\frac{\alpha_s}{\pi}G\,G\right\rangle &= 0.024\,\pm\,0.012\,\mathrm{GeV}^4\,,\\
\langle\overline{q}\sigma Gq\rangle &= m_0^2\,\langle\overline{q}q\rangle\,, \qquad \text{with } m_0^2 = 0.8\,\pm 0.2\,\mathrm{GeV}\,,\\
m_s(2\,\mathrm{GeV}) &= 120\,\pm 50\,\mathrm{MeV}\,,\\
\langle\overline{s}s\rangle &= (0.8 \pm 0.3)\langle\overline{q}q\rangle\,.
\end{aligned} \tag{3.4}$$

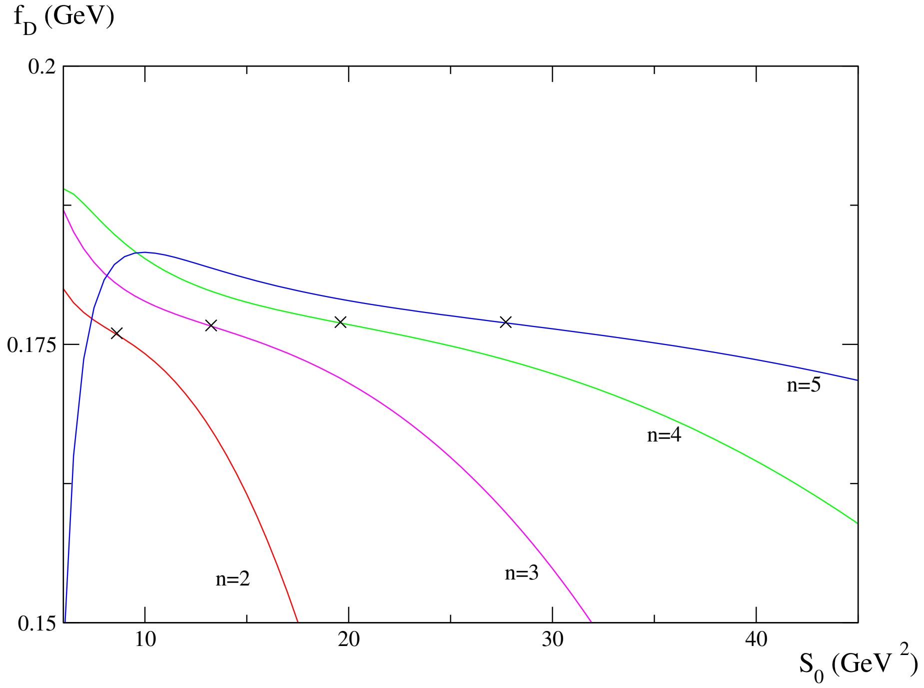

Figure 1: Decay constant f_D as a function of the integration radius s_0 for $m_c(m_c) = 1.25\,\mathrm{GeV}$. The lines represent the sum rule (2.4) for several choices of the polynomial $P_n(s)$.

As discussed above, we fix the renormalization scale to be $\mu = m_c(m_c)$. We use a reasonable variation of μ to analyze the corresponding uncertainty in our final result. Finally, for the charm quark, we take the value advocated by the Particle ata Group $m_c(m_c) = 1.25 \pm 0.10\,\mathrm{GeV}$ [28]. Using a more recent average value $m_c(m_c) = 1.28 \pm 0.10\,\mathrm{GeV}$ [29] would lower the central value of our result for f_D and f_{D_s} by 5 MeV.

In order to calculate the decay constant for the pseudoscalar meson D, we proceed in the way described above. We computef_D as a function of s_0 with the four different sum rules (2.4) corresponding to $n = 2, 3, 4, 5$. The results, plotted in figure 1 show remarkable stability properties. We define the optimal value of s_0 as the center of the stability region (represented by a cross in figure 1) where the first and/or second derivative of $f_D(s_0)$ vanishes. At these values of s_0 we obtain the following consistent results:

$$\begin{aligned} f_D &= 176\,\mathrm{MeV} \qquad \text{for } n = 2 \\ f_D &= 177\,\mathrm{MeV} \qquad \text{for } n = 3, 4, 5\,. \end{aligned} \tag{3.5}$$

Notice from figure 1 that for the fifth degree polynomial ($n = 5$) there is a stability region of about $20\,\mathrm{Ge}V^2$ around s_0, where the decay constant changes by less than three percent. From this change we estimate a conservative error inherent to the method of $\pm 3\,\mathrm{MeV}$. We consider the small dependence of our results on the parameter s_0 as one of the main advantages of our method. For instance, in earlier papers eg. [30] in which decay constants of the heavy pseudoscalar mesons are estimated, the duality parameter s_0 is fixed by comparing two different sum rules, but the question of stability in s_0 is not even addressed.

Inputs	$f_D = 177\,\mathrm{MeV}$
$m_c(m_c) = 1.25\ \pm 0.10\,\mathrm{GeV}$	$-19 + 9\,\mathrm{MeV}$
$\langle\overline{q}q\rangle(2\,\mathrm{GeV}) = (-267\ \pm\ 17\,\mathrm{MeV})^3$	$\pm 5\,\mathrm{MeV}$
$\left\langle\frac{\alpha_s}{\pi}GG\right\rangle = 0.024\ \pm\ 0.012\,\mathrm{GeV}^4$	$\pm 1\,\mathrm{MeV}$
$\langle\overline{q}\sigma Gq\rangle\ = (\,0.8\ \pm 0.2\,\mathrm{GeV})^2\,\langle\overline{q}q\rangle$	$\pm 1\,\mathrm{MeV}$
$\alpha_s\,(m_c) = 0.335 \pm 0.010$	$\pm 3\,\mathrm{MeV}$

Table 1: Uncertainties on f_D from the uncertainties on the theoretical parameters.

Other sources of errors arising in the calculation of f_D are the quark condensates which affect the result by $\pm 5\,\mathrm{MeV}$, and the charm mass which, in the range given above, produces a variation in the decay constant of $-19, +9\,\mathrm{MeV}$. This is one of the main source of uncertainty in the final result (see table 1). Considering the following results of the perturbative α_s expansion:

$$\begin{aligned} \text{one-loop calculation}\ :\ \ f_D^{(0)} &= 142\,\mathrm{MeV} \\ \text{two-loop calculation}\ :\ \ f_D^{(1)} &= 162\,\mathrm{MeV} \\ \text{three-loop calculation}\ :\ \ f_D^{(2)} &= 177\,\mathrm{MeV} \end{aligned} \tag{3.6}$$

we also take as a source of uncertainty the contribution of order α_s^2, which amounts ten percent of the result. This yields an asymptotic error of $\pm 15\,\mathrm{MeV}$.

We point out that the convergence of the asymptotic series in the present calculation of the decay constant of the D meson is worse than the one we found for the B meson [25]. Finally we have considered the dependence on the renormalization scale in the range $\mu \in [2, 6]\,\mathrm{GeV}$. The error associated to this change in μ is roughly related to the convergence of the asymptotic expansion and therefore it is not considered as an additional one. Other errors due to the QCD side of the sum rule, higher order terms in m_c^2/s and the error on $\alpha_s(m_Z)$, are negligible in comparison.

From this analysis of errors, we finally quote the following result for the decay constant of D-meson

$$f_D = 177\ \pm\ 14\,(\mathrm{inp.})\ \pm 15(\mathrm{asymp.}) \pm\ 3\,(\mathrm{meth.})\,\mathrm{MeV}\,. \tag{3.7}$$

The first error comes from the inputs of the computation, the second to the truncated QCD theory whereas the last one is due to the method itself.

Proceeding in the same fashion, but keeping the mass of the strange quark in the overall factor and the order m_s/m_c in the one loop contribution, we find the decay constant for the D_s meson,

$$\begin{aligned} \text{one-loop calculation}\ :\ \ f_{D_s}^{(0)} &= 163\,\mathrm{MeV} \\ \text{two-loop calculation}\ :\ \ f_{D_s}^{(1)} &= 188\,\mathrm{MeV} \\ \text{three-loop calculation}\ :\ \ f_{D_s}^{(2)} &= 205\,\mathrm{MeV} \end{aligned} \tag{3.8}$$

and including the analysis of uncertainties we find:

$$f_{D_s} = 205\ \pm\ 14\,(\mathrm{inp.})\ \pm 17(\mathrm{asymp.}) \pm\ 3\,(\mathrm{meth.}))\,\mathrm{MeV}\,,$$

(see figure 2).

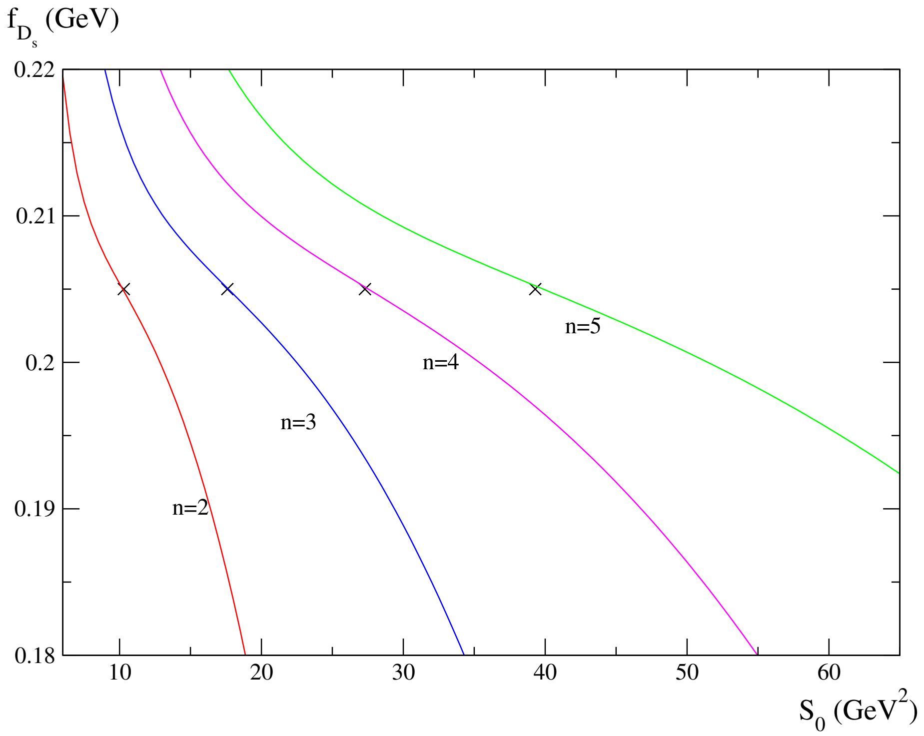

Figure 2: Decay constant f_{D_s} as a function of the integration radius s_0 for $m_c(m_c) = 1.25\,\text{GeV}$. The lines represent the sum rule (2.4) for several choices of the polynomial $P_n(s)$.

In the analysis of theoretical errors the only new ingredient is the uncertainty coming from the strange quark mass which turns out to be negligible.

The ratio of the decay constants f_{D_s} and f_D (which would be 1 in the chiral limit) is of special interest. We find:

$$\frac{f_{D_s}}{f_D} = 1.16 \,\pm\, 0.01\,(\text{inp.}) \,\pm\, 0.03\,(\text{meth.})\,. \tag{3.9}$$

in complete agreement with lattice calculations. The uncertainties due to the theoretical inputs are correlated, so that the final error is very small.

4. Conclusions

In this note we have computed the decay constant of D_q-mesons for q being either the strange or the u or d massless quarks. We have employed a judicious combination of moments in QCD finite energy sum rules in order to minimize the shortcomings of the available experimental data. On the theoretical side of the pseudoscalar two-point function, we have used in the perturbative piece an expansion up to three loops in the strong coupling constant and up to order $\left(m_c^2/s\right)^7$ in the mass expansion and in the non-perturbative piece we considered condensates up to dimension six including the α_s correction in the leading term. Instead of the commonly adopted pole mass of the bottom quark, we use the running mass to improve convergence of the perturbative series.

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In the sum rule, the contour integration of the asymptotic part is performed analytically. This particular fact differs from other computations based on sum rules where the asymptotic QCD is integrated along a cut of the two-point function starting at the pole mass squared. The latter way to proceed is problematic when loop corrections are included and the complete analytical QCD expression along the cut is not known. In this approach QCD has to be extrapolated from low energy to high energy [30]. We also differ from many other sum rule calculation in that we do not require two unrelated sum rules to determine a duality point via an intercept of the curves $f_D(s_0)$.

Our results are very sensitive to the value of the running mass, giving most of the theoretical uncertainty. They also turn out to be sensitive to variations of the renormalization scale μ. The uncertainties of other theoretical parameters like quark condensates and coupling constant are less important. Adding quadratically the different estimated errors we have the final results

$$\begin{aligned} f_D &= 177 \pm 21\,\mathrm{MeV} \\ f_{D_s} &= 205 \pm 22\,\mathrm{MeV}\,. \end{aligned} \tag{4.1}$$

Comparing (4.1) with other results in the literature, our results agree within the error bars with the ones obtained using sum rule methods [10, 34]. However, compared with lattice computations [35, 36] they are a bit lower.

A. Orthogonal polynomials

For convenience of the reader we list in this appendix the first few polynomials emerging from relations (2.11), (2.12). From the second condition, namely (2.12), it is easy to realize that the set of polynomials $P_n(s)$ are n-degree orthogonal polynomials in the interval of the variable $s \in [s_{\mathrm{cont.}}, s_0]$. With the normalization condition (2.11) (adopted to emphasize the contribution of the lowest lying resonance in the sum rule) they are related to the Legendre polynomials $\mathcal{P}_n(x)$ in the interval of the variable $x \in [-1, 1]$ as follows:

$$P_n(s) = \frac{\mathcal{P}_n\left(x(s)\right)}{\mathcal{P}_n\left(x(M_{D_q}^2)\right)}\,. \tag{A.1}$$

Where the variable $x(s)$ is:

$$x(s) = \frac{2s - (s_0 + s_{\mathrm{cont.}})}{s_0 - s_{\mathrm{cont.}}}\,.$$

Obviously $x(s) \in [-1, 1]$ when $s \in [s_{\mathrm{cont.}}, s_0]$.

The explicit form of these polynomials is well known and can be found, for instance, in [37]. Nevertheless, for sake of completeness, we quote here the ones we have used in the calculation.

$$\begin{aligned} \mathcal{P}_2\left(x(s)\right) &= \frac{1}{2}(3x^2 - 1)\,, \\ \mathcal{P}_3\left(x(s)\right) &= \frac{1}{2}(5x^3 - 3x)\,, \end{aligned}$$

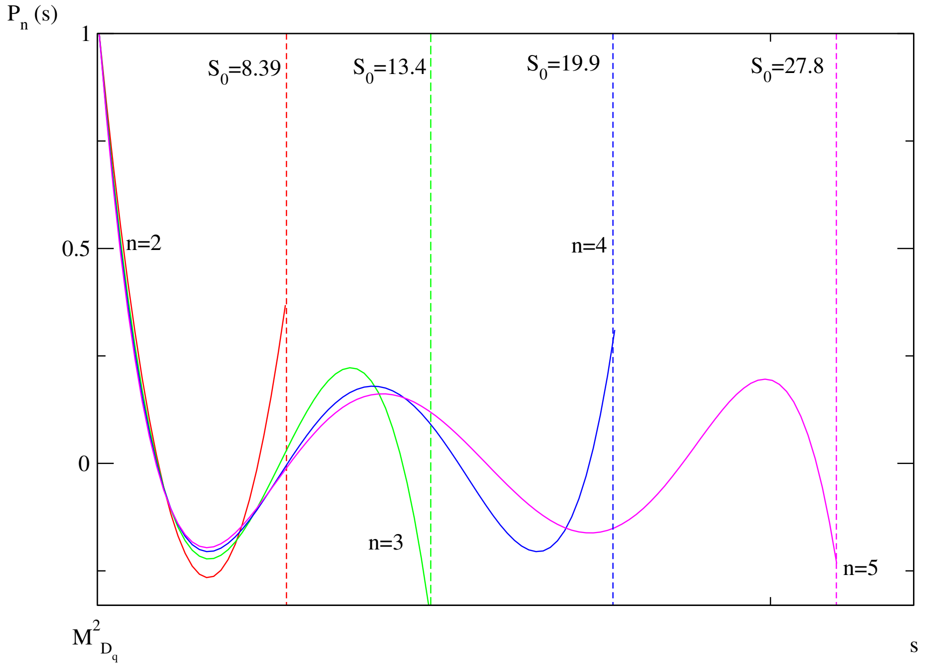

Figure 3: Polynomials $P_n(s)$ obtained from conditions (2.11), (2.12) taken at the stability values of s_0 in the calculation of f_B.

$$\mathcal{P}_4\left(x(s)\right) = \frac{1}{8}(35x^4 - 30x^2 + 3)\,,$$
$$\mathcal{P}_5\left(x(s)\right) = \frac{1}{8}(63x^5 - 70x^3 + 15x)\,.$$

Finally, in figure 3 and in order to appreciate the suppression of the experimental physical continuum data in the sum rule, we have plotted the form of the polynomials $P_n(s)$ for $n = 2, 3, 4, 5$ at the stability values of s_0 used in the calculation of f_D.

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